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SPHERES, HEMISPHERES AND DISCS AS HIGH-PERFORMANCE FINS FOR BOILING HEAT TRANSFER

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Abstract—The prediction of the performance of high-performance spines for heat transfer to boiling liquids can be calculated logically by means of the local assumption for the heat transfer coefficient. The maximum heat duty is predicted adequately by a 1-dimensional conduction analysis. Accuracy in predicting the accompanying base temperature favors a 2-dimensional conduction model. Fins with discontinuities in the surface shape require trial and error for optimization. A 3-disc fin has been developed which exceeds the performance of the best cylinder by a factor of 4·9 and the turnip shape by a factor of 3·6

NOMENCLATURE

C_n,	integration constant;
1-*D*,	one dimensional;
2-*D*,	two dimensional;
h,	local heat transfer coefficient;
k,	thermal conductivity of solid;
P_n,	Legendre polynomials of order n;
Q,	heat transfer rate;
r,	radial distance to an internal point;
R,	outer radius;
T,	local temperature;
T_B,	temperature at fin base (wall temperature);
T_L,	saturation temperature of liquid;
ΔT,	$T - T_L$;
ΔT_B,	$T_B - T_L$;
ε,	stem angle defined in Fig. 2;
ϕ,	azimuthal angle defined in Fig. 2.

INTRODUCTION

THE MAIN difficulty in designing fins for use in boiling liquids arises from the fact that the heat transfer coefficient is not constant. Thus the constant-h formulas in wide use for applications involving convection with no change of phase are not applicable. During boiling, the heat transfer coefficient is a function of the surface-to-liquid ΔT. Therefore, the variable-h formulas based on a stated functionality between h and position [1–4] are not applicable.

The "local assumption" described by Haley and Westwater [5, 6] has proved to be a good method of handling the variable h. The local heat transfer coefficient at a point on a fin is assumed to be exactly the same as exists on an isothermal surface having the same temperature. This method has beeome widely adopted [7–17]. Particularly noteworthy is a recent study [11] which obtained experimentally the local values of h and ΔT and showed that the local assumption is quite reasonable.

It is of interest to select the optimum shape for fins used in boiling liquids. If the fin is a cylinder of constant diameter, and if one-dimensional (1-*D*) conduction may be assumed, the design method is documented [6]. Two-dimensional (2-*D*) conduction in a cylindrical fin was first calculated [18] for the case of a length-to-diameter ratio of 5. These 2-*D* results were almost the same as those for 1-*D*.

For a fin of circular cross section but a varying diameter, the problem of optimization is much more difficult. For the 1-*D* case, a method has been published [6]. If the base temperature is hot enough to cause film boiling, the optimum spine has the shape of a turnip. The reason the turnip shape is efficient is that it has a large surface area where the heat transfer coefficient is large and a small surface area where the h is small. The stem of the turnip serves to decrease the temperature from a high value at the wall to a value corresponding to a high heat transfer coefficient.

A significant objection to the turnip shape is that it is difficult to manufacture. However, the complex shape can be approximated by simple shapes. One approximation [7] consists of two cones, base-to-base, connected to a stem. A large number of the double-cone fins have been computed. Three have been tested experimentally and were found to yield higher heat duties than predicted, presumably because the predictions were based on 1-*D* rather than 2-*D* conduction. Some of the double-cone fins had a length-to-diameter ratio near unity, and one would suspect that the 1-*D* analysis might show deviation at this aspect ratio. Inasmuch as a sphere also has a

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length–diameter ration of unity, it was selected for study by 1-*D* and 2-*D* computations and by experimentation. A hemisphere, a disc on a stem, and multiple discs were also investigated. The object of this study was to develop a fin shape of superior performance—superior to the turnip shape which was an optimization for 1-*D* conduction.

SPHERE ON A STEM

Mathematical 1-dimensional

Figure 1 shows a sphere and stem constructed of copper. The 1-*D* mathematical model considers the conduction equations in the direction only of the geometric axis. This is equivalent to assuming that the thermal conductivity of the metal is infinite in directions normal to this axis. For analysis, the sphere is subdivided into 24 incremental slices, and each is assumed to be of uniform temperature. The governing equations are readily solved on a digital computer using a finite difference technique.

In the present case, full advantage is taken of the simplifications resulting from spherical symmetry. Figure 2 defines the symbols.

Equation (1) is the governing equation for the solid sphere.

$$r^2 \cdot \frac{\partial^2 T}{\partial r^2} + 2r\frac{\partial T}{\partial r} + \frac{1}{\sin\phi} \cdot \frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial T}{\partial\phi}\right) = 0. \quad (1)$$

Assuming a constant thermal conductivity, it has the following conditions:

B.C. 1 at $\phi = 0$, $\dfrac{\partial T}{\partial\phi} = 0$

B.C. 2 at $\phi = \pi$, $\dfrac{\partial T}{\partial\phi} = 0$

B.C. 3 at $r = R$, $0 \leqslant \phi < \pi - \varepsilon$,

$$\frac{\partial T}{\partial r} + \frac{h}{k} \cdot T = \frac{h}{k} T_L \quad (2)$$

B.C. 4 at $r = R$, $\pi - \varepsilon \leqslant \phi \leqslant \pi$ $T = T_B(\phi)$. (3)

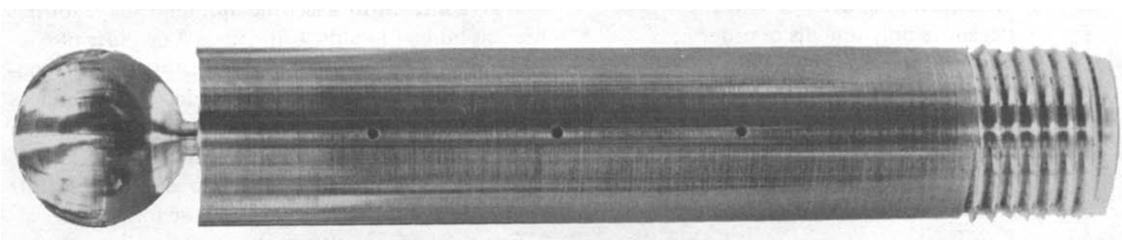

FIG. 1. Spherical fin of 1·22 cm dia on a 2·44 mm dia stem mounted on a supporting rod which can be screwed into an electric heater.

Mathematical 2-dimensional

The finite difference technique can be used for the 2-*D* case. However, a superior, iterative, point-matching technique was developed and is described below. Prior applications of the point-matching method have been described by others, e.g. [19–21].

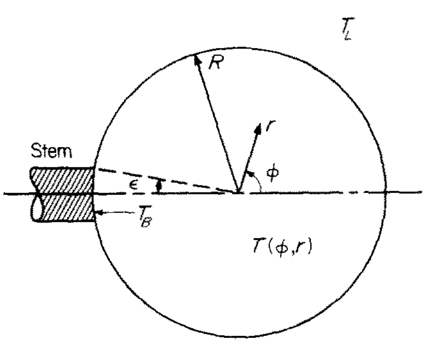

FIG. 2. Sketch of spherical fin on a stem.

The first two conditions result from symmetry, and the third is a heat balance at the surface. The fourth is troublesome, because the junction between the sphere and the stem is not isothermal and it does not have a uniform heat flux [22]. The difference in temperature from $\phi = \pi - \varepsilon$ to $\phi = \pi$ in the stem was calculated by a 1-*D* analysis, then this temperature difference was evenly distributed over the curved interface between the sphere and the stem.

Equation (1) is a partial differential equation. Assuming the temperature to be a product of two functions depending on ϕ and r respectively, then equation (1) can be separated into a Legendre equation and an Euler equation. The associated eigen values should be integers $1, 2, \ldots$ in order that the solution be finite at both $\phi = 0$ and π. The temperature can be expressed in an infinite series:

$$T = \sum_{n=0}^{\infty} C_n P_n(\cos\phi)\left(\frac{r}{R}\right)^n. \quad (4)$$

The experimental numerical values of h were introduced to correspond to the local surface temperatures. A point-matched short trial-and-error iteration scheme was employed to evaluate the constants C_n's. The infinite series in equation (4) was truncated to 40 terms. Trials with 60 terms [22] justified this cutoff. Substituting equation (4) in equations (2) and (3) and selecting 5 points within the stem and 35 points on the surface, a system of 40 equations was obtained in the following form.

Within the neck, $i = 1, \ldots, 5$

$$\sum_{n=0}^{39} P_n(\cos\phi_i)\, C_n = T_B(\phi_i) \tag{5}$$

on the surface, $i = 6, \ldots, 40$

$$\sum_{n=0}^{39} \left[\frac{h(\phi_i)}{k} + \frac{n-1}{R}\right] C_n = \frac{h(\phi_i)}{k} \cdot T_L. \tag{6}$$

By solving the system for the C_n's, the temperature at any point can be calculated according to equation (4). The total heat duty is found by summing over the surface

$$Q = \int_0^{\pi-\varepsilon} \int_0^{2\pi} h(\phi)(T - T_L)\, R^2 \sin\phi \, \mathrm{d}\theta \, \mathrm{d}\phi. \tag{7}$$

Figure 3 shows the results for a copper spherical fin of 1·22 cm dia on a stem of 2·44 mm dia in boiling Freon-113 (CCl_2F-$CClF_2$). The read-in values of the h vs ΔT functionality used were those obtained experimentally in the authors' laboratory and published previously as graphs [6, 23].

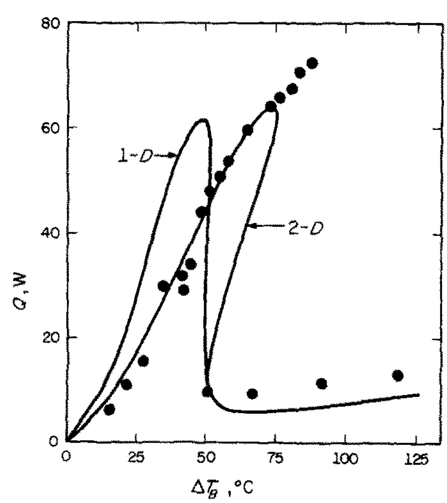

FIG. 3. Comparison of 1-dimensional and 2-dimensional predictions with data for spherical fin of Fig. 1. Freon-113 on copper at atmospheric pressure.

It is interesting that the peak heat duties as found by the 1-D and 2-D mathematical curves are nearly equal. Calculations for 5 other sphere diameters, 3 other stem diameters, and 2 other solid materials, in isopropanol as well as in Freon-113 suggest that this observation is general [22]. The slopes of the curves are different for the 1-D and 2-D predictions, with the peak heat duty occurring at a greater ΔT_B for the 2-D case. In Fig. 3, the predicted peak heat duties differ by only 5 per cent, but the predicted ΔT_B values corresponding to these peaks differ by 35 per cent.

Experimental

The equipment used to test the spines was the same as that used by Haley and Westwater [6]. It consisted of a 4 l. stainless steel boiler with glass windows. Vapor rose to an overhead condenser. The fins and their supporting rod were inserted into a copper slug containing two 350 W electrical heaters. The fin was sealed in the wall of the boiler with a Teflon gasket.

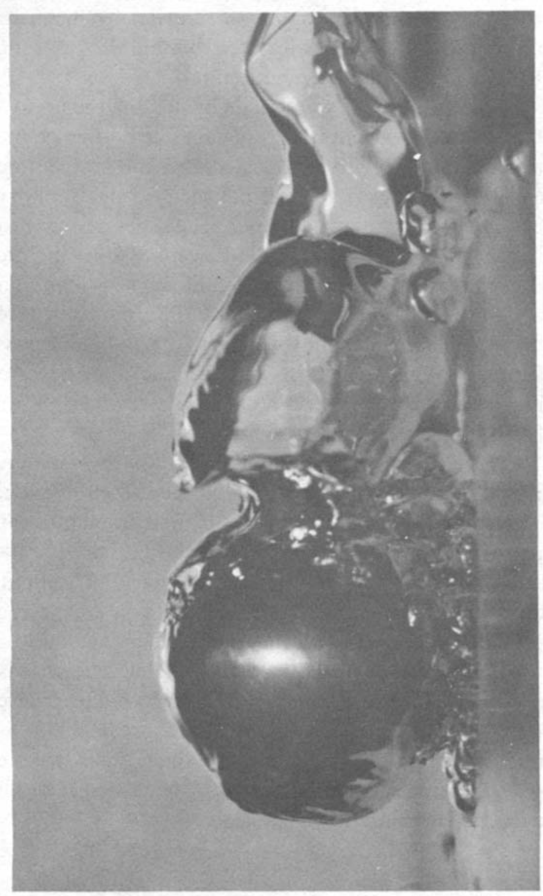

FIG. 4. Spherical fin in boiling isopropanol. Film boiling covers the entire fin. $Q = 28$ W and $\Delta T_B = 124$°C.

Table 1. Comparison of various copper fins optimized for Freon-113. Each fin assembly had a volume of 0·963 cm^3

	Max. Q (W)		ΔT_B, °C for max. Q		Surface	Projected
	Calculated	Exptl.	Calculated	Exptl.	area (cm^2)	area (cm^2)
Cylinder	43·7	48	91·7	90	6·78	0·316
Turnip	58·6	80	91·7	155	6·41	0·845
Double-cone	74·4	78	111·0	183	5·76	1·580
Sphere 1-D	61·5		47·2	89	4·65	1·208
Sphere 2-D	64·5	73	75·0	89	4·65	1·208
Hemisphere	72·6	—	30·0		5·54	1·850
1-Disc	153·0	187	122·2	148	15·79	7·29
2-Disc	181·7	218	113·9	143	16·72	4·05
3-Disc	213·9	270	127·8	169	21·37	3·86
4-Disc	211·0		127·8		23·23	3·60
5-Disc	193·4	—	127·8		28·80	4·30

In the supporting rod were three thermocouples on the geometric axis. The temperature of the fin base was computed by extrapolating the temperatures in the supporting rod. Heat duties were determined by three methods: (a) by calculating the temperature gradient in the supporting rod, (b) by making a heat balance around the condenser, and (c) by metering the condensate. They all agreed within 10 per cent. Results are reported by method (a).

Film boiling runs were made by heating the fin to a temperature high enough to sustain film boiling before adding liquid. This is the "dry charge" method. Before each test the fin surface was polished by means of dry 3/0 emery paper and a commercial copper cleaner in a consistent way. The test liquids were distilled before use. Data were taken on different days for each fin and showed good reproducibility. Photographs were taken by use of a photoflash at 10^{-6} s, which was short enough to freeze any bubble motion. An illustration of the sphere in use is Fig. 4, which shows film boiling over the entire fin.

Figure 3 shows experimental results for a sphere. It is obvious that the data follow the 2-D prediction. The agreement is excellent for the first branch of the performance curve where nucleate boiling predominates. It is not as good for the third branch (film boiling), for which the data lie about 60 per cent above the prediction. The middle branch of the curve cannot be obtained experimentally when simple electric heat is used. The observed peak of the performance curve was about 14 per cent higher than predicted. This might mean that some modest amendment to the read-in values of the local h vs ΔT is needed. The fin data are summarized in Table 1.

HEMISPHERE ON A STEM

A primary reason for using fins during heat transfer is to gain more surface area. A solid hemisphere has 19 per cent more surface area than a sphere of equal volume. A 1-D conduction model of a hemisphere having a volume of 0·963 cm^3 (the same fin volume used for the fin in Figs. 1 and 3) was solved. The orientation was with the circular flat face perpendicular to the geometric-axis of the stem and opposite the stem. This orientation required minimal changes in the computer program. The peak heat duty for copper in Freon-113 was calculated to be increased 18 per cent above that for the equal-volume sphere. This demonstrated clearly the important role of surface area and indicated that attempts to increase the area could be rewarding provided the resistance to conduction inside the fin be controlled properly. This led to the disc designs which are presented herein.

DISCS ON A STEM

Single-disc

Figure 5 illustrates the 2-step, 1-dimensional, computation model used for one disc on a stem. The heat flow was assumed to be 1-D axially in the stem and 1-D radially in the disc. A finite-difference method was used with 15 nodes in the disc. Considering the previous fin volume, 0·963 cm^3, the best radius was

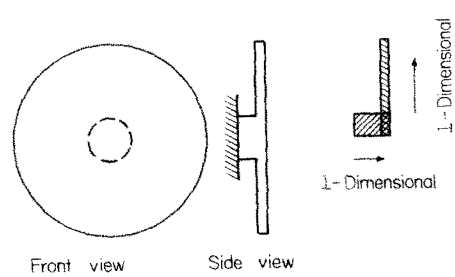

FIG. 5. Sketch of 2-step, 1-dimensional computational model for a disc on a stem.

found by trial and error to be 1·524 cm, for copper in Freon-113. The dimensions are recorded in Table 2. This gave a peak heat duty which was 2·6 times the maximum for the turnip shape. This means that the optimization procedure used by Haley and Westwater, which was modified slightly from the procedure used earlier by Wilkins [24], is applicable only for 1-*D* conduction in a fin having no discontinuities in its radius. At present no mathematical technique is known for optimizing a fin, taking into consideration 2-*D* conduction and discontinuities.

Table 2. Dimensions of the copper disc fins

	Thickness (mm)	Diameter (cm)
1-disc	1·270	3·048
2-disc	1·270	2·088
	1·016	2·312
3-disc	1·016	1·880
	0·889	2·012
	0·699	2·270
4-disc	0·889	1·626
	0·889	1·626
	0·635	1·920
	0·508	2·144
5-disc	0·762	1·540
	0·762	1·540
	0·572	1·778
	0·445	2·012
	0·330	2·336

In all cases the stem segments were 3·18 mm long and 6·35 mm dia. Each fin assembly in this table had a volume of 0·963 cm^3.

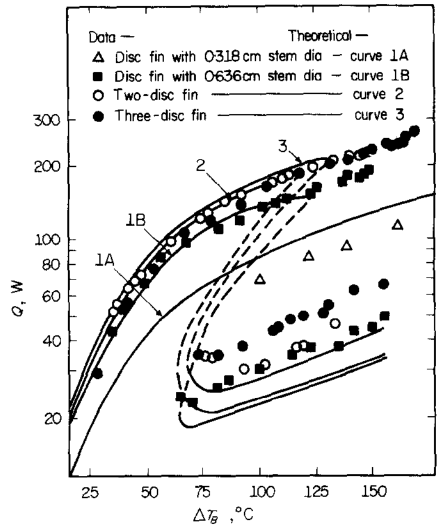

FIG. 6. Predicted curves and experimental data for 1, 2 and 3-disc copper fins in boiling Freon-113.

The predicted performance curve for the single-disc fin is represented in Fig. 6 by Curves 1A and 1B for two stem diameters. Changes in the stem dimensions shift the curve to the left or right but do not affect the maximum heat duty. Here the maximum duty is 153 W. Experimental checks of these two boiling curves were carried out, and the laboratory data are shown in Fig. 6. The agreement is reasonable, particularly in view of the fact that 2-*D* computations were avoided. The measured peak heat duty for the disc on the larger stem was 23 per cent above the predicted value.

A single-disc fin of a second metal, aluminum, was tested in Freon-113. This fin had the same projected area and the same weight, but not the same volume as the single copper disc of Fig. 6. The results are shown in Fig. 7, along with the results for the equal-mass copper disc. The dimensions of the aluminum fin are given below Fig. 7. The peak heat duty for the aluminum fin, as found by experimentation as well as by prediction, is significantly greater than the peak for the copper fin. The aluminum fin is 3 times as thick and has 15 per cent more surface area than the copper fin. The proper read-in values of h vs ΔT for aluminum are not known, thus they were assumed to be the same as for copper. This may explain why the agreement between prediction and data for the aluminum fin are the poorest of all cases in this paper. Along the main branch of the performance curve, the measured heat duty was about 20 per cent below the predicted values; however, the data

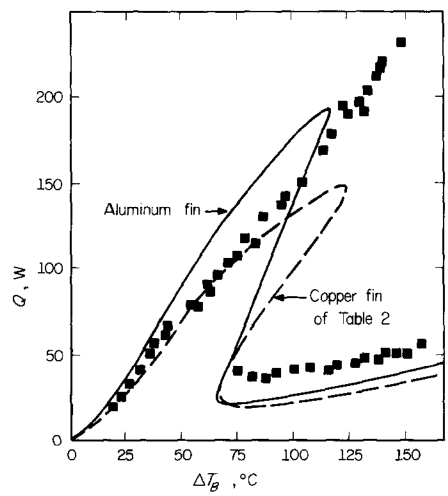

FIG. 7. Predicted curve and experimental data for 1-disc aluminum fin in Freon-113. Disc diameter was 3·05 cm, thickness was 0·39 cm, stem diameter was 0·85 cm, stem length was 0·159 cm.

go to a higher ΔT_B and finally give a measured maximum heat duty 20 per cent above the predicted peak.

The stem design used throughout this investigation was simple. The stem serves as a resistance to decrease the temperature from the value at the wall to a value corresponding to a high heat transfer coefficient. The heat dissipated from the stem to the surrounding liquid is insignificant. The stem diameter and length are selected arbitrarily to provide the desired thermal resistance. The stem diameter needs to be large enough to satisfy strength requirements. The stem length needs to be great enough (say 1 or 2 mm) to permit the easy escape of bubbles from the stem, the back side of the fin, and the wall surrounding the stem. There is appeal in the use of a disc having internal tapped threads mounted on a threaded stud for a stem. This permits easy adjustment of the stem length. One such device was constructed, a 1·8 cm dia copper disc, 3 mm thick, mounted on a copper stud having 16 threads per cm. This was tested, in Freon-113, with the effective stem length being varied. The predicted and measured peak heat duties agreed nicely. However, the measured ΔT_B was far in excess of the predicted values, signifying that the thermal resistance at the metal metal junction was serious.

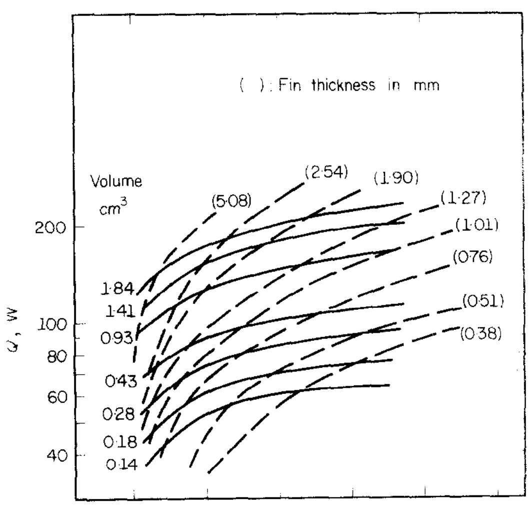

FIG. 8. Effect of disc size on peak heat duty and base temperature difference for 1-disc copper fins in Freon-113. The stem diameter was constant, 6·35 mm. For this figure only, ΔT_B refers to the junction between the disc and the stem.

For a solid disc of fixed volume, the peak heat duty can be increased theoretically without limit by increasing the disc radius and simultaneously decreasing the thickness. However, the increase in duty is accompanied by an increase in the temperature required at the base of the fin. Figure 8 is a family of curves showing the peak duty for fins of different volume (solid lines). The disc thickness is a parameter, and from the thickness one may readily calculate the disc radius. The largest disc included in this graph has a diameter of 4·35 cm, and the smallest is 1·33 cm.

In practical applications, a multiple number of fins are used, spaced as closely together as conditions permit. A disc of large diameter is objectional, because it has a large projected area and restricts the number of fins which can be located on a wall of given area. A disc of tapered thickness was considered. A triangular profile (thickest at the base and sharp at the outer edge) was found to be superior to a trapezoidal profile or a rectangular (constant thickness) profile. However, the improvement in heat duty was only 8–15 per cent, depending on fin thickness, and this was less than the increase in projected area.

A cup-shaped fin design was considered also. The stem was attached to the center of the bottom of the cup. The wall of the cup was assumed to have a constant thickness. By trial it was found that a typical shallow cup can reduce the projected area by 30 per cent compared to a disc, while suffering only a 2 per cent decrease in peak heat duty. A deep cup fin can reduce the projected area by 55 per cent at the expense of a 10 per cent decrease in heat duty.

An alternate idea for increasing surface area without increasing the radius of a disc is to cut radial slots in the disc. Of course, the area increase is insignificant if the disc is very thin, but it may be substantial for a thick disc. For example, a solid copper disc of 2·38 cm dia and 2·54 mm thickness had a predicted maximum duty of 132 W in Freon-113. Consider a slotted disc with the same volume: namely a disc of 2·18 cm dia, 3·8 mm thick, that has 20 radial slots, each 0·8 mm wide and 4·6 mm long. The predicted maximum heat duty for this is 200 W at the same ΔT_B of 44·5°C. This 52 per cent improvement in heat duty compares to the 57 per cent increase in surface area. The possible use of slotted fins, tapered fins, and cups was not pursued far. The use of multiple discs on a stem was found to be vastly superior, as will be described.

Multiple discs

Figure 8 may be used to compare single discs of various sizes. In particular, the peak heat duty per unit volume is of interest. On this basis, small fins are superior to large ones. The advantage of multiple discs on a stem is obvious. This permits a high heat duty per unit volume and maintains a small projected area. The wall temperature must be high enough to cause film boiling, and the stem segments must be designed to maintain film boiling on their surfaces. ment is calculated, and the dimensions of the parts are adjusted so that all discs reach the peak heat flux simultaneously. Details of the trial and error design method are available [22]. The largest (and thinnest) disc is nearest the wall, and the smallest disc is farthest away.

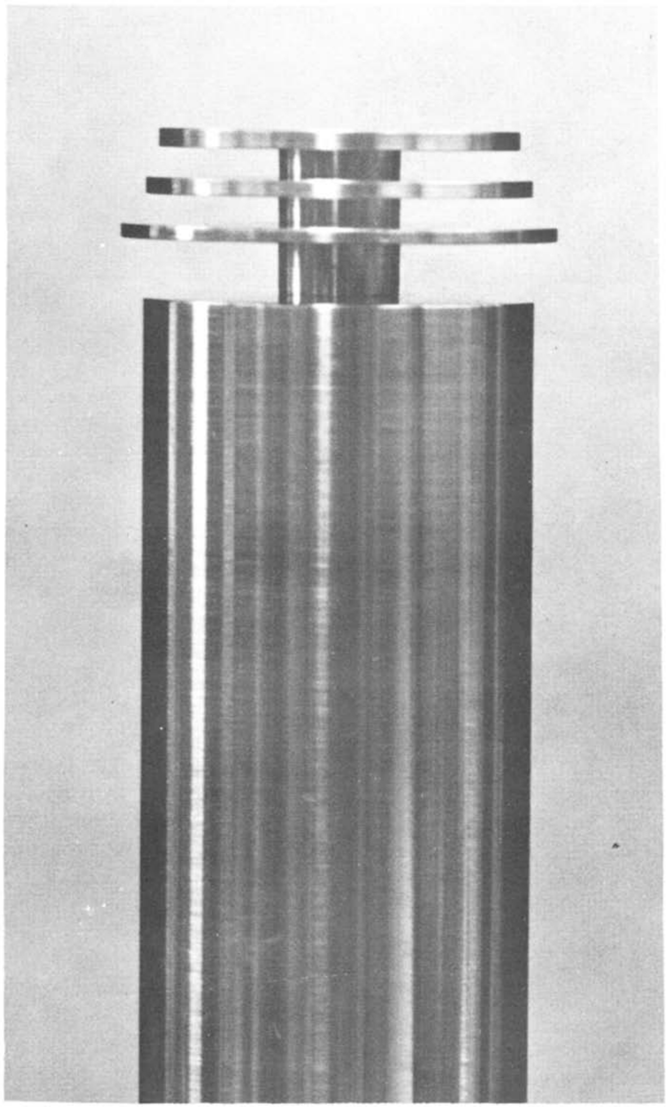

FIG. 9. Three-disc fin on a stem mounted on a supporting rod, side view. Dimensions in Table 2.

The clearance between discs should be the smallest which permits rapid escape of the bubbles. This is about 1·6 mm according to prior studies [25]. The performance curve for each disc is calculated separately, the temperature drop across each stem seg-

Table 2 shows the results of multiple disc designs, carried out as described, for copper in Freon-113. The volume of each fin assembly is 0·963 cm^3. There is no guarantee that any of these dimensions are true optima, although enough trials were computed to convince the writers that these are near optima. Figure 9 is a side view of the 3-disc fin. Figure 10 is a front view of the same fin in use in boiling Freon-113. The predicted performance curves for these assemblies (with the exception of the 4- and 5-disc models) are given in Fig. 6. Also included are laboratory data for the 1-, 2-, and 3-disc models. For the main branch of the performance curves, the agreement between

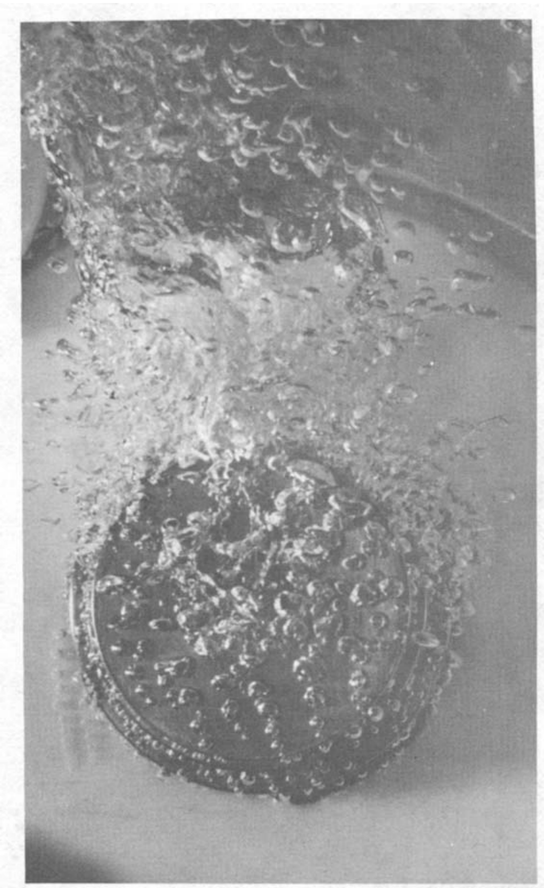

FIG. 10. Front view of 3-disc copper fin in boiling Freon-113. $Q = 98$ W and $\Delta T_B = 66{\cdot}5$°C.

predictions and measurements is good. The superiority of 2 and 3 discs over a single disc is evident.

Table 1 is a summary comparison for all the shapes with the same volume. The 4-disc model gave essentially the same performance as the 3-disc model, but the 5-disc model showed a decrease in performance. The reason for the existence of an optimum number of discs is that some metal must be used in the stem segments and this reduces the metal in the discs.

A second liquid, isopropanol, was used to test the 3-disc fin and the sphere. The experimental data

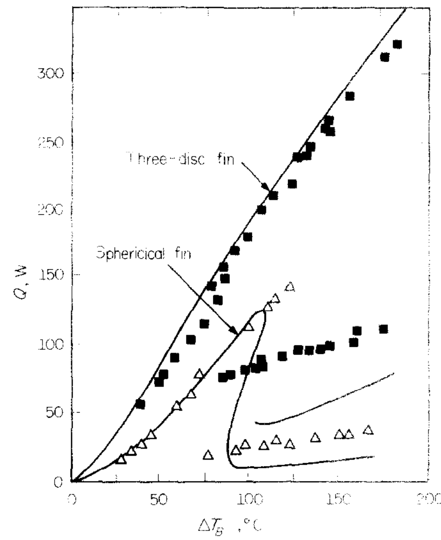

FIG. 11. Predicted curves and experimental data for 3-disc and spherical copper fins in isopropanol. The predicted peak heat duty was 405 W at 220°C. The highest data point was obtained using the limiting heat output, which was not sufficient to reach the peak.

and the predicted curves are given in Fig. 11. The agreement between experiments and predictions for the main branch of the performance curves is good. The superiority of the multiple disc model over the sphere of equal volume is obvious.

CONCLUSIONS

Fins which are discontinuous shapes with large surface area, such as multiple discs on a stem, are superior to continuous shapes such as cylinders, spheres, and turnips. The design of the discontinuous shapes requires an intuitive approach. This means that additional improvement may be possible. Experiments with two metals and two liquids substantiate the design methods.

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SPHÈRES, HEMISPHÈRES ET DISQUES COMME RUGOSITÉS Á HAUTE PERFORMANCE POUR LE TRANSFERT THERMIQUE PAR ÉBULLITION

Résumé—Le calcul des performances des rugosités à haute performances pour le transfert thermique aux liquides bouillants peut être effectué logiquement à partir de l'estimation du coefficient de transfert local. Le flux maximal de chaleur est évalué correctement par une analyse de conduction monodimensionnelle. Le calcul précis de la température de base nécessite un modèle de conduction à deux dimensions. Des rugosités avec des discontinuités de surface doivent être étudiées par une méthode d'approximations successives pour atteindre l'optimum. On a considéré une rugosité à trois disques qui présente des performances 4,9 fois supérieures à celles du meilleur cylindre et 3,6 fois supérieures à celle d'un bulbe.

KUGELN, HALBKUGELN UND SCHEIBEN ALS HOCHLEISTUNGSRIPPEN FÜR DEN WÄRMEÜBERGANG MIT VERDAMPFUNG

Zusammenfassung—Die Berechnung des Wirkungsgrades von Hochleistungsrippen für den Wärmeübergang an siedende Flüssigkeiten kann mittels der Annahme eines lokalen Wärmeübergangskoeffizienten erfolgen. Die maximale Nutzleistung wird entsprechend durch ein 1-dimensionales Wärmeleitungsberechnungsverfahren bestimmt. Die Genauigkeit bei der Bestimmung der auftretenden Rippengrundtemperatur wird durch ein 2—dimensionales Wärmeleitungsmodell begünstigt. Rippen mit Unstetigkeiten in der Oberflächenform erfordern für die Optimierung eine Näherungsrechnung. Es wurde eine 3—Scheibenrippe entwickelt, die den Wirkungsgrad der besten Zylinderrippe um einem Faktor 4,9 und die Rübenform um einer Faktor 3,6 übersteigt.

ИСПОЛЬЗОВАНИЕ СФЕРИЧЕСКИХ, ПОЛУСФЕРИЧЕСКИХ И ДИСКОВЫХ РЕБЕР ДЛЯ ПОВЫШЕНИЯ ЭФФЕКТИВНОСТИ ТЕПЛООБМЕНА ПРИ КИПЕНИИ

Аннотация—Проведен расчет рабочих характеристик высокоэффективных ребер для интенсификации теплоотдачи к кипящим жидкостям, в котором учитывались значения локального коэффициента теплообмена. Максимальный режим рассчитан с помощью одномерного анализа теплопроводности. Более точные значения сопутствующей основной температуры получены с помощью двумерной модели теплопроводности. В случае разрывных поверхностей ребер оптимальный расчет получается с помощью метода проб и ошибок. Разработано ребро из трех дисков, эффективность которого в 4,9 раза больше эффективности цилиндрического ребра и в 3,6 раза больше эффективности ребра в форме луковицы.